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Improving the Convergence of SU(3) Baryon Chiral Perturbation Theory

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Abstract

Baryon chiral perturbation theory as conventionally applied using dimensional regularization has a well-known problem with the convergence of the SU(3) chiral expansion. One can reformulate the theory equally rigorously using a momentum-space cutoff and we show that the convergence is thereby greatly improved for reasonable values of the cutoff. In effect, this is accomplished because the cutoff formalism removes the spurious physics of propagation at distances much smaller than the baryon size.

1 Basic ideas

Chiral perturbation theory describes low energy QCD via a simultaneous expansion in the energy and in quark masses.[1, 2] The original application to the physics of pions and kaons has enjoyed a remarkable success and the convergence of the energy expansion has been acceptable for realistic applications. The formalism in the baryon sector has also been developed,[3, 4, 5, 6, 7], but here one finds problems with the convergence of the chiral expansion, especially in the application of chiral SU(3)—typically, SU(3) symmetry relations are well satisfied at tree-level (i.e. to first order in the quark masses), but when “improved” to one-loop the corrections are larger than are allowed by experiment. Well known examples include

- i) Baryon masses, wherein the Gell-Mann-Okubo relation obtains at first order in quark mass and is well satisfied experimentally. Chiral loops, however, make significant—O(50-100%)—corrections to individual masses.[8]
- ii) Semileptonic hyperon decay, wherein a simple SU(3) representation of the axial couplings in terms of F,D couplings yields an excellent fit to experiment. Chiral loops make O(30-50%) corrections to individual couplings and destroy this agreement.[9]
- iii) Nonleptonic hyperon decay, wherein a simple SU(3) fit to s-wave amplitudes provides an excellent representation of the experimental numbers in terms of f,d couplings. Chiral loops make O(30-50%) corrections to individual terms and destroy this agreement. The situation is somewhat more confused in the case of the p-waves wherein a significant cancellation between pole terms exists at lowest order and the validity of the chiral expansion is suspect.[10]

Of course, these loop modifications can in general be rescued by including yet higher orders in the chiral expansion, but the net result is that the expansion is not well behaved in that there is no clear convergence to the orders yet calculated.

In this paper, we show that this SU(3) violation from loop diagrams arises to a large extent from propagation at short distances—smaller than the physical size of baryons—where the effective field theory cannot properly represent the correct physics. If we remove this short-distance physics

by the application of a cutoff, we obtain a greatly improved phenomenology and a nicely convergent expansion. Chiral perturbation theory can be formulated equally rigorously with a cutoff as with the conventionally employed dimensional regularization,[11] and we will demonstrate how this works in a specific example below. We shall show that the formalism of baryon chiral perturbation theory with a cutoff then opens the possibility for useful SU(3) chiral phenomenology in the baryon sector.

Effective field theory is a technique which uses the very low energy interactions and degrees of freedom of a theory in order to calculate the long distance physics appropriate for low energy problems.[12] The effects of short-distances/high-energies are not calculated directly but are encoded in the coefficients of a general local effective Lagrangian. When loop diagrams are calculated, the reliable portion of the result comes only from the long-distance/low-energy portion of the loop, as that is the only portion for which the effective theory is appropriate. In the same diagrams, there are always also contributions from high energy which are not correctly represented by the effective theory. However, this is not a problem in principle because these spurious high energy contributions are equivalent to terms in the local effective Lagrangian and can be corrected for by a shift in coefficients of this Lagrangian. The formalism then allows one to calculate rigorously the long distance dynamics of loop processes while providing a general parameterization of the short distance physics.

In the interaction of pions with baryons, the long distance physics is the propagation of pions out to large distances—the so-called “pion tail”. Because the pion is light its propagator has significant strength at distances of order of its Compton wavelength, *i.e.* ~ 1.4 fermis. The effective field theory formalism represents baryons and pions as point particles, even though we know that they have a non-zero size. This is not a problem for the lowest energy propagation—the pion tail is model independent. However, this feature tells us where “short distance” physics starts. The effective theory in terms of point particles misrepresents the physics on distance scales smaller than the actual size of the particles. For example, a baryon has a rms charge radius of ~ 0.8 fermis, so that loop effects below this distance are not reliably calculable in the effective theory of point baryons.

Chiral loops do in fact generate large contributions from short distance—in general they are divergent. The most common regularization scheme to avoid this problem is dimensional regularization, which is elegant and simple

to apply. In such a scheme, in addition to the correct long distance behavior, there is a residual dependence on the short distance portion of the loop integrals. It is also possible to regularize the loop integrals by a momentum-space cutoff.[11] This effectively removes the short distance propagation. Either scheme can be used *equally rigorously* in a chiral effective field theory, as long as care is taken to preserve the chiral symmetry. Since the schemes differ only in the treatment of the short distance portion of the theory, they will involve different coefficients of higher order terms in the chiral expansion. However, as long as these are treated fully generally, the same physics must result.

We will demonstrate below how the use of a cutoff can be applied in baryon chiral perturbation theory and how it resolves the problem with the convergence of the energy expansion. A longer paper will present full details and additional examples of this reformulation of the theory,[13] while here we concentrate simply on the underlying physics. First we demonstrate how to reproduce known results in baryon masses using the cutoff formalism. This includes several non-trivial consistency checks. Subsequently, by choosing a realistic value of the cutoff, representing the onset of short distance physics, we show how the problematic (and apparently unphysical) symmetry breaking effects generated by loop contributions are moderated in this formalism.

2 Baryon masses

An example of the difficulties of baryon chiral perturbation theory is provided by the analysis of baryon masses. The masses have an expansion in the masses of the quarks (m_q), or equivalently in terms of the pseudoscalar meson masses (m_M)

$$M_B = M_0 + \Sigma_q \bar{b}_q m_q + \Sigma_q \bar{c}_q m_q^{3/2} + \Sigma_q \bar{d}_q m_q^2 + \dots \quad (1)$$

$$= M_0 + \Sigma_M b_M m_M^2 + \Sigma_M c_M m_M^3 + \Sigma_M d_M m_M^4 + \dots \quad (2)$$

Here M_0 is a common mass and b_M and d_M contain adjustable parameters representing terms in the effective Lagrangian. However, the non-analytic $m_q^{3/2}$ terms come from loop diagrams, and the coefficients are not adjustable but are known in terms of the baryon-meson coupling constants. The leading SU(3) breaking terms involving b_M go back to Gell-Mann and Okubo[14], the

non-analytic corrections from one-loop diagrams, represented above by c_M , were first calculated by Langacker and Pagels[15], and m_M^4 corrections (including diagrams up to two loops) were calculated by Borasoy and Meissner[8]. The convergence difficulties in the expansion are demonstrated by the resulting fit for the nucleon mass where, in the same sequence, the different contributions are given, in GeV, by[8]

$$M_N = 0.711 + 0.202 - 0.272 + 0.298 + \dots \quad (3)$$

or, more dramatically for the Ξ ,

$$M_\Xi = 0.767 + 0.844 - 0.890 + 0.600 + \dots \quad (4)$$

The non-analytic terms appear unavoidably large and the expansion has certainly not converged at this order. The final fit also violates the Gell-Mann-Okubo relation by an amount that is five times larger than the experimentally observed violation.

To one-loop order, the explicit form of the contributions to the baryon masses is given by[7]

$$\begin{aligned} M_N &= \hat{M}_0 - 4m_K^2 b_D + 4(m_K^2 - m_\pi^2) b_F + L_N \\ M_\Lambda &= \hat{M}_0 - \frac{4}{3}(4m_K^2 - m_\pi^2) b_D + L_\Lambda \\ M_\Sigma &= \hat{M}_0 - 4m_\pi^2 b_D + L_\Sigma \\ M_\Xi &= \hat{M}_0 - 4m_K^2 b_D - 4(m_K^2 - m_\pi^2) b_F + L_\Xi \end{aligned} \quad (5)$$

where

$$\hat{M}_0 = M_0 - 2(2m_K^2 + m_\pi^2) b_0 \quad (6)$$

with M_0 , b_D , b_F and b_0 as free parameters. (Note that M_0 and b_0 do not have separate effects, but only enter in the combination \hat{M}_0 .) The ingredients L_B contain the nonanalytic contributions from loop diagrams, and have the form[16]

$$\begin{aligned} L_N &= -\frac{1}{24\pi F_\pi^2} \left[\frac{9}{4}(D+F)^2 m_\pi^3 + \frac{1}{2}(5D^2 - 6DF + 9F^2) m_K^3 + \frac{1}{4}(D-3F)^2 m_\eta^3 \right] \\ L_\Lambda &= -\frac{1}{24\pi F_\pi^2} \left[3D^2 m_\pi^3 + (D^2 + 9F^2) m_K^3 + D^2 m_\eta^3 \right] \end{aligned}$$

$$\begin{aligned}
L_\Sigma &= -\frac{1}{24\pi F_\pi^2} \left[(D^2 + 6F^2)m_\pi^3 + 3(D^2 + F^2)m_K^3 + D^2m_\eta^3 \right] \\
L_\Xi &= -\frac{1}{24\pi F_\pi^2} \left[\frac{9}{4}(D - F)^2m_\pi^3 + \frac{1}{2}(5D^2 + 6DF + 9F^2)m_K^3 + \frac{1}{4}(D + 3F)^2m_\eta^3 \right]
\end{aligned} \tag{7}$$

where D and F parameterize the baryon axial-vector current ($D + F = 1.266$ and $D/(D + F) = 0.64$). The non-analytic terms are quite large, having values

$$\begin{aligned}
L_N &= -0.31 \text{ GeV}, \\
L_\Lambda &= -0.66 \text{ GeV}, \\
L_\Sigma &= -0.67 \text{ GeV}, \\
L_\Xi &= -1.02 \text{ GeV}.
\end{aligned} \tag{8}$$

using $D = 0.806$ and $F = 0.46$ and $F_\pi = 93\text{MeV}$. (The slight numerical disagreement with the fit quoted in Eq. 3,4 occurs because the authors of Ref. 3 used somewhat different D, F and F_π values.) In particular, the Ξ mass shift is clearly unphysically large. It is not possible to obtain a reasonably convergent fit to the masses with these large non-analytic terms to this order.

3 Regularization with a cutoff

The mass analysis is especially simple in the heavy baryon formalism[4, 5, 6] using dimensional regularization where all of the mass shifts are proportional to a single integral

$$I(m_P^2) = \frac{1}{i} \int \frac{d^4k}{(2\pi)^4} \frac{\mathbf{k} \cdot \mathbf{k}}{(k_0 - i\epsilon)(k^2 - m_P^2 + i\epsilon)} \tag{9}$$

where m_P is the mass of the Goldstone boson that is involved in the loop. When dimensionally regularized, this has results in

$$I(m_P^2) = \frac{m_P^3}{8\pi} \tag{10}$$

Observe that a peculiarity of dimensional regularization is that, although this integral appear cubically divergent via power-counting, the dimensionally

regularized form is finite even for $d \rightarrow 4$. There is also the counter-intuitive feature that the result vanishes in the massless limit, wherein we would expect the long distance physics to be the most important, and also grows larger for the more massive states than for the pion, while physically we would expect the reverse. It appears that a short-distance subtraction is implicit in this formalism. Since the meson mass-squared is proportional to the quark mass, this integral is uniquely the source of the non-analytic terms in the previous results, L_B .

Let us now calculate this integral with a momentum space cutoff.[16] There are many forms which could be employed equivalently, and we chose one possibility with an eye to the finite spatial size of the baryons. Since the heavy baryon defines a preferred frame of reference—the rest frame—we may choose a form which regulates only the spatial momentum components in that frame. In covariant notation this is, *e.g.*, a cutoff of the form

$$e^{\frac{k^2 - (v \cdot k)^2}{\Lambda^2}} \sim e^{-\frac{\mathbf{k}^2}{\Lambda^2}} \quad (11)$$

where v_μ is the unit four-vector that defines the rest frame of the baryon. With this regularization the integral becomes

$$I(m_P^2) = \frac{1}{8\pi} \left[\frac{\Lambda^3}{2\sqrt{\pi}} - \frac{\Lambda m_P^2}{\sqrt{\pi}} + m_P^3 e^{\frac{m_P^2}{\Lambda^2}} \left(1 - \Phi\left(\frac{m_P}{\Lambda}\right) \right) \right] \quad (12)$$

where $\Phi(x)$ is the probability integral

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x dt e^{-t^2} \quad (13)$$

When the meson mass is small compared to the cutoff, the integral simplifies to

$$I(m_P^2) \xrightarrow{m_P^2 \ll \Lambda^2} \frac{1}{8\pi} \left[\frac{\Lambda^3}{2\sqrt{\pi}} - \frac{\Lambda m_P^2}{\sqrt{\pi}} + m_P^3 \right] \quad (14)$$

Let us demonstrate how the usual results are recovered in this limit. The Λ^3 contribution yields simply an overall shift to the baryon mass, of the same form as M_0 , while the Λm_P^2 contribution has the same form as the first SU(3) breaking terms. Thus the results at this order have the generic form

$$M_B = (M_0 + k\Lambda^3) + \sum (b_P - k\Lambda)m_P^2 + \sum c_P m_P^3 \quad (15)$$

The dependence on Λ can be completely absorbed into renormalized values of M_0 and b_i . To accomplish this in practice in fact requires highly non-trivial consistency checks. There are four baryon masses, and each mass expression contains the loop integral for pions, kaons and etas. However, there is only a single M_0 , and three b_i that must absorb all dependence on Λ . Thus there are numerous consistency requirements if this is to work properly. One can check that all are satisfied and that all Λ dependence is completely absorbed via the definitions

$$\begin{aligned}
M_0^{ren} &= M_0 - \frac{5D^2 + 9F^2}{24\pi F_\pi^2} \Lambda^3 \\
b_D^{ren} &= b_D + \frac{3F^2 - D^2}{64\pi F_\pi^2} \Lambda \\
b_F^{ren} &= b_F + \frac{5DF}{96\pi F_\pi^2} \Lambda \\
b_0^{ren} &= b_0 + \frac{13D^2 + 9F^2}{288\pi F_\pi^2} \Lambda
\end{aligned} \tag{16}$$

This is a verification that our cutoff regularization respects the chiral symmetry, as expected. Thus when the meson masses are small compared to the cutoff, the usual analysis is completely reproduced.

On the other hand, when the meson mass becomes large compared to the cutoff, the effect of the loop diagram is moderated. For example, if the mass is large compared to the cutoff, we have the result

$$I(m_P^2) = -\frac{3}{32\pi^{3/2}} \frac{\Lambda^5}{m_P^2} \ , \tag{17}$$

i.e. it vanishes for large m_P . This is physically reasonable, as for large mass only a vanishingly small portion of the loop integral occurs at low momentum. Note that the dimensionally regularized form of the integral does not allow this distinction—the result grows continuously with increasing meson mass and there is no separation of the short and long distance components.

When employing any regularization scheme that introduces a dimensionful parameter, the usual power counting rules will be upset. This is manifest in the results quoted above, in which the lowest order chiral parameter, M_0 is shifted by the loop correction. However, since these shifts are just the renormalization of phenomenological parameters, they do not influence

the physics, and a proper chiral expansion of the final results will always be obtained. In practice, renormalization with a cut-off is not much more difficult than that in the framework of dimensional regularization.

Our results do not depend on the specific form of the cutoff function employed. For example, we have also considered a dipole cutoff

$$\left(\frac{\Lambda^2}{\Lambda^2 - k^2}\right)^2, \quad (18)$$

which produces the same qualitative features with the replacement

$$I(m_P^2) = \frac{1}{8\pi} \left[\frac{\Lambda^4}{(\Lambda^2 - m_P^2)^2} (m_P^3 - \Lambda^3) + \frac{3}{2} \frac{\Lambda^5}{\Lambda^2 - m_P^2} \right] \quad (19)$$

and the renormalization of the chiral parameters is effected in the same way.

4 Phenomenology

The formalism may be applied with any value of the cutoff, as long as the cut-off is not chosen so small that it removes physics that is truly long distance. However, if the cutoff is chosen too large, the convergence may be poor because the loop calculation will include spurious short-distance physics which will have to be removed by counterterms at higher order in the energy expansion. The best values of the cutoff are then those close to the scale where the effective field theory description starts to be inaccurate.

In baryons, the physical size of the hadrons is ~ 1 fermi. For propagation over distances much below this scale, the effective field theory description in terms of point baryons and pions will no longer be accurate. Therefore we want to choose a cutoff representative of that scale. In the case of the electromagnetic interaction, the baryon-photon vertex is known to have a dipole shape with a mass scale around the rho mass. The self energy diagram involves two vertices, so that mimicking the effects of form-factors would suggest a quartic shape, leading to an estimate of $\Lambda \sim m_\rho/2$. Note, however, that strictly speaking we are not doing field theory with form-factors, but are merely regularizing the loop integrals with a cutoff. In practice, any cutoff around this value will be sufficient.

We note that the kaon and eta masses are not sufficiently small compared to the cutoff that all of their effects need be long-distance. Employing a

	$\Lambda = 300$	$\Lambda = 400$	$\Lambda = 500$	$\Lambda = 600$
\bar{I}_π	0.00611	0.0157	0.03198	0.0567
\bar{I}_K	0.0024	0.0077	0.01833	0.0363
\bar{I}_η	0.0020	0.0069	0.01633	0.0329

Table 1: Given are numerical values of the integral $I(m_P^2)$ (Eq. 9) in GeV^3 for various values of the cutoff Λ given in MeV. For comparison, dimensional regularization of this integral corresponds to the values 0.00246, 0.1213 and 0.1815 for π , K and η respectively.

reasonable cutoff will keep only the long distance portion of the loop diagram. We then may use the loop integral directly in the phenomenology. For the baryon masses, this leaves the previous formulas of Eq. 7 unchanged except for the substitution in the non-analytic terms of the form

$$m_P^3 \rightarrow \bar{I}(m_P^2) \quad (20)$$

with

$$\bar{I}(m_P^2) = m_P^3 e^{\frac{m_P^2}{\Lambda^2}} \left(1 - \Phi\left(\frac{m_P}{\Lambda}\right) \right) + \frac{\Lambda^3}{2\sqrt{\pi}} - \frac{\Lambda m_P^2}{\sqrt{\pi}} \quad (21)$$

In Table 1, we show that this substitution significantly moderates the magnitude of the non-analytic terms for kaons and etas for any reasonable value of the cutoff. This by itself is a demonstration that much of the kaon and eta loop integrals, when calculated dimensionally, actually correspond to short-distance physics and are not reliable parts of the chiral effective field theory. As is physically reasonable, the pion has the largest long-distance loop effect, and the value decreases as the meson becomes more massive. Note that in this instance we are not absorbing the Λ into the chiral parameters, but are keeping the cutoff in the full loop effect, $\bar{I}(m_P^2)$.

In Table 2, we show the loop correction to the baryon masses for various values of Λ . These are much smaller than the corresponding results in dimensional regularization. In addition, since these results contain an overall shift in M_0 , one also notes that the SU(3) breaking is decreased greatly, yielding very reasonable amounts of SU(3) breaking. For any of these values

	dim.	$\Lambda = 300$	$\Lambda = 400$	$\Lambda = 500$	$\Lambda = 600$
N	-0.31	-0.04	-0.11	-0.22	-0.40
Σ	-0.67	-0.03	-0.08	-0.18	-0.34
Λ	-0.66	-0.03	-0.08	-0.18	-0.34
Ξ	-1.02	-0.02	-0.06	-0.15	-0.29

Table 2: Given (in GeV) are the nonanalytic contributions to baryon masses in dimensional regularization and for various values of the cutoff parameter Λ in MeV.

of Λ , we can obtain an excellent fit to the baryon masses. There is not much dynamical content in such a fit, as the loop effects are now small enough that we cannot demonstrate the presence of them in the data. However, we see an excellent description without the need for yet higher orders. The convergence of the expansion is now much improved.

5 Summary

The procedure described above matches well with the goals of effective field theory because it keeps only the long-distance portion of loop diagrams. It turns out that only a portion of kaon and eta loops are truly long-distance, and that inclusion of just these effects yields a chiral expansion that is well behaved.

We now understand the origin of the previous problem with the convergence of the chiral expansion in baryons. The previous analyses using dimensional regularization had implicitly included spurious short-distance physics in the loop calculation. This required correction at higher orders in the energy expansion, and appeared to lead to a poor convergence. The cutoff regularization excludes this spurious physics, and therefore leads to a better description.

It is certainly true that it is more difficult, and occasionally more subtle [11, 13], to work with a momentum space cutoff than with the usual dimensional procedure. Indeed in the meson sector the dimensional formal-

ism works fine. However, in baryons the improved convergence properties obtained by use of a cutoff justifies the extra effort. Indeed, this development may finally allow realistic phenomenology to be accomplished in SU(3) baryon chiral perturbation theory.

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